ofoils and Wings," *Transonic Aerodynamics*, AGARD Conference Proceedings 35, 1968, pp. 11-1 to 11-23.

[2]Mager, A., "On the Model of the Free, Shock-Separated, Turbulent Boundary Layer," *Journal of the Aeronautical Sciences*, Vol. 23, Feb. 1956, pp. 181–184.

[3]Smith, D. K., "Wind Tunnel Investigation of the Pressure Distribution on a Two-Dimensional Airfoil with Pylon Mounted Stores at Mach Numbers from 0.70 to 0.90," AEDC-TR-73-71, March 1973, Arnold Engineering Development Center, Arnold Air Force Station, Tenn.

[4]Krupp, J. A., "The Numerical Calculation of Plane Steady Transonic Flows Past Thin Lifting Airfoils," PhD. thesis, June 1971, University of Washington, Seattle, Wash.

[5]Studwell, V. E., "Investigation of Transonic Aerodynamic Phenomena for Wing Mounted External Stores," PhD. thesis, June 1973, University of Tennessee Space Institute, Tullahoma, Tenn.

# Longitudinal Response of an Aircraft due to a Trailing Vortex Pair

A. Ramirez Jr.,* B. M. Rao,† and A. E. Cronk‡
*Texas A&M University, College Station, Texas*

A survey of the available literature on prediction of airloads on aircraft due to trailing vortices of another aircraft and subsequent dynamics of the aircraft subjected to such loads revealed a limited amount of information. Research in this area has been limited due to the complexity in adequately describing the vortex formation, motion, and dissipation. Furthermore, flight test data would be almost impossible to obtain due to the apparent danger imposed on the aircraft and its crew.

In this study, an approximate method to predict the response of a small aircraft in longitudinal motion when it is flying through a trailing vortex system of a large aircraft is developed. In the analysis, the vortex system was idealized as two parallel line vortices in a horizontal plane. The following aircraft was in horizontal flight at a fixed distance from the plane of the vortices and did not perturb the vortices. Several other simplifying assumptions were made: 1) controls are fixed, 2) coupling effects between the lateral and longitudinal motions are neglected, 3) spinning rotor effects are neglected, and 4) effect of the aircraft on the trailing vortices is neglected.

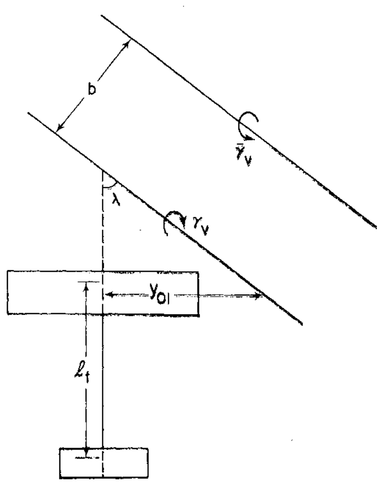


Fig. 1 Aircraft-vortex pair representation.

Received May 15, 1973; revision received July 30, 1973.
Index category: Aircraft Handling, Stability, and Control.

*Graduate Assistant, Aerospace Engineering.
†Associate Professor, Aerospace Engineering. Member AIAA.
‡Professor and Head, Aerospace Engineering. Associate Fellow AIAA.

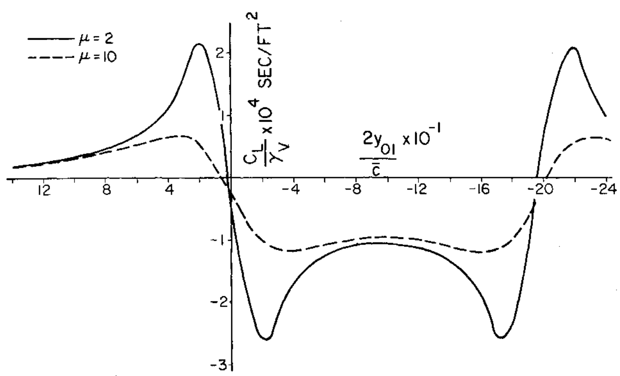


Fig. 2 Variation of incremental lift coefficient per unit vortex strength with position of left vortex.

Table 1 Dimensions and specifications of De Havilland Beaver

| | |
|---|---|
| C.G. location | |
| Aft L.E. MAC | 35.0% MAC |
| Below L.E. MAC | 36.0 in. |
| Max gross weight for this condition | 4800 lbs |
| Areas | |
| Wing, total | 250 ft² |
| Horizontal tail, total, approx. | 43.82 ft² |
| Dimensions and general data | |
| Wing span | 48 ft |
| Horizontal tail span | 15.83 ft |
| Wing chord (MAC) | 5.2 ft |
| Horizontal tail chord (MAC), approx. | 2.76 ft |
| Dihedral | 2° |
| Wing aspect ratio | 9.2 |
| Tail aspect ratio, approx. | 5.71 |
| Distance from wing MAC quarter chord point to horizontal tail MAC quarter chord point ($l_t$) | 19.76 ft |

The longitudinal stability equations of motion were linearized as in Ref. 1 by the application of the small disturbance theory and the external aerodynamic loads were assumed to consist of two parts. The first part was due to the presence of the trailing vortex pair and the second part was due to the changes in the flight variables which are conventionally represented in terms of the stability derivatives. The analysis was applied to a particular small

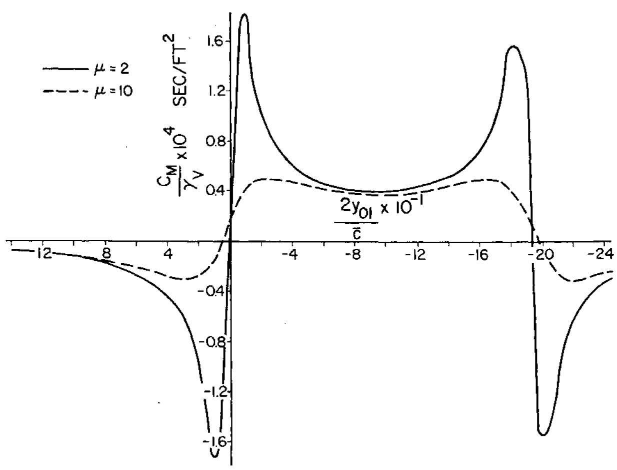


Fig. 3 Variation of incremental pitching moment coefficient per unit vortex strength with position of left vortex.

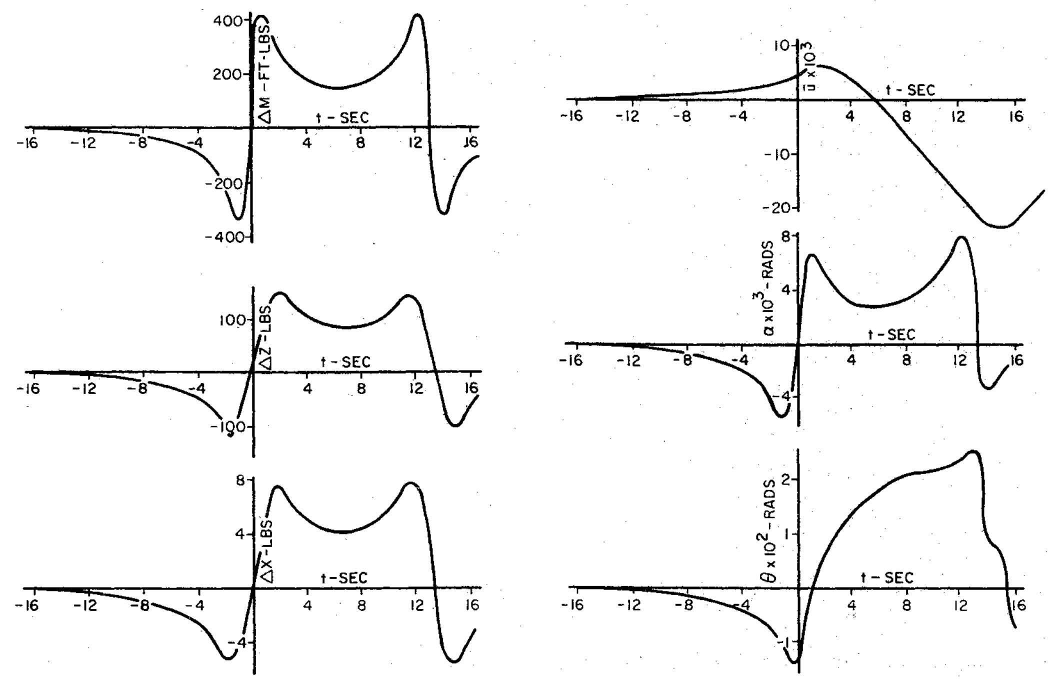


**Fig. 4 Time history of flight variables for a vortex strength of 100 ft $^2$/sec and a height of one chord length.**

aircraft, the De Havilland Beaver, and the strength of the trailing vortex pair was assumed to be small so that the linearization of the equations of motion is justified.

The aerodynamic loads induced by the presence of the vortex pair were completed by modifying the lifting surface technique developed by Jones and Rao.[2] In this technique, the vortex distribution on the lifting surface is assumed to be a finite double Fourier trignometric series and the downwash distribution produced by the pair of trailing vortices is known. The solution to the integral-differential equation between the downwash and the vortex distributions on the lifting surface was reduced to a set of algebraic equations by selecting a finite number of chordwise and spanwise stations. The solution of these algebraic equations yield the coefficients of the finite double Fourier series for the vortex distribution. Then the aerodynamic forces and moments were calculated from the known vortex distribution. In this analysis, the loads were computed assuming the wing and the horizontal tail as the lifting surfaces and neglecting the wing-body-tail interference effects. The details of the analysis were given in Ref. 3.

The specification of the test aircraft were given in Table 1 and the idealized model used to compute the aerodynamic loads was shown in Fig. 1. The test aircraft was assumed to be at an altitude of 5000 ft ($\rho$ = 0.002 slugs/ft$^3$) with a flight velocity of 170.6 fps, and at an angle $\lambda$ = 5°. The span of the trailing vortex system was assumed to be that of a Boeing 747, 195 ft, and was assumed to be at a height of one and five chord lengths ($\mu$ = 2, and $\mu$ = 10) from the test aircraft. The results of the lift and moment coefficient variations with respect to the distance between the test aircraft and the left vortex ($y_{01}$) were shown in Figs. 2 and 3. The distance, $y_{01}$, is directly related to the flight speed of the aircraft and the angle of incidence, $\lambda$, of the trailing vortex pair. For the test aircraft, the stability derivatives and the drag polar were given in Ref. 4.

The longitudinal dynamic stability analysis was performed for the aircraft when the aircraft penetrated the trailing vortex pair of strength 100 ft$^2$/sec and at $\lambda$ = 5°. Although the vortex strength of Boeing 747 is considerably larger than the assumed strength, the value was chosen to keep the problem linear. This can be justified in view of the fact that the vortex strength decreases exponentially and the chosen strength may be considered as the strength at several thousand feet away from the large aircraft. Time histories of the flight variables $\bar{u}$, $\alpha$, and $\theta$, which represent the changes in flight speed, angle of attack, and pitching angle respectively, were shown in Fig. 4. Our stability analysis is analogous to an arbitrary system which is subjected to a forced oscillation. The forced oscillation term is analogous to the input forces and moments due to the trailing vortex system. When the forces and moments due to the trailing vortex system are zero, the problem reduces to describing the motion of the aircraft due to small disturbances. In this note, the lateral response is not studied. In a subsequent study, the same problem is considered and the complete aircraft response is studied treating it as a 6 degree-of-freedom system.

## References


[1]Etkins, B., *Dynamics of Flight,* Wiley, New York, 1958, Chap. 4, pp. 94–144.

[2]Jones, W. P. and Rao, B. M., "Airloads and Moments on an Aircraft Flying Over an Inclined Trailing Vortex," *Aircraft Wake Turbulence,* Plenum Publishing Corp., New York, 1971, pp. 523–545.

[3]Ramirez, A. H., "A Study of an Aircraft Subjected to the Airloads by a Pair of Trailing Vortices of a Large Aircraft," M. S. thesis, Dec. 1971, Texas A&M Univ. College Station, Tex.

[4]Gerlach, O. H., "Determination of Performance and Stability Parameters from Nonsteady Flight Test Manuevers," Rept. 700234, March 1970, Society of Automotive Engineers, New York.